

Supplementary Material for “Evolution Matters: Content Transmission in Evolving Wireless Social Networks”

Some proofs and discussions as well as experimental results are omitted in the main paper for readability, and we provide the missing content in this supplementary material for completeness.

I. MISSING PROOFS

A. Proof of Lemma 1

We prove the conclusion by two parts: (i) $G(V, E)$ is connected $\Leftrightarrow B(V, I)$ is connected. (ii) $B(V, I)$ is connected $\Leftrightarrow B_0(V_0, I_0)$ is connected.

(i) For the first case, to prove the forward implication, we first focus on the connectivity between users V in $B(V, I)$. Since $G(V, E)$ is connected, for any two distinct nodes u and v in $G(V, E)$, there exists a path connecting them. We assume the path to be $u, w_1, w_2, \dots, w_k, v$. For any edge in this path, say (w_j, w_{j+1}) , according to the relationship between $G(V, E)$ and $B(V, I)$, users w_j and w_{j+1} must have a same neighbor i_j in $B(V, I)$. Thus, we can convert the path in $G(V, E)$ to a path in $B(V, I)$ connecting u and v , which is $u, i_0, w_1, i_1, w_2, i_2, \dots, i_{k-1}, w_k, i_k, v$ (for any k , $w_k \in V$, $i_k \in I$). Therefore, for any two nodes connected in $G(V, E)$, they must be connected in $B(V, I)$ as well.

For any two nodes both from I in $B(V, I)$, since they both have at least c_i neighbors in $B(V, I)$, we can find two neighbors in V connecting them respectively (the case that the two neighbors turn out to be the same is trivial). We only need to examine the connectivity between them, which has been discussed above.

Following similar argument, it is enough to prove the case where two nodes come from V and I respectively.

Conversely, since $G(V, E)$ and $B(V, I)$ share the same node set, for any two distinct nodes $u, v \in V$ in the bipartite graph $B(V, I)$, there exists a path $u, i_0, w_1, i_1, w_2, i_2, \dots, i_{k-1}, w_k, i_k, v$ connecting them. Recall the generation of $G(V, E)$ from $B(V, I)$, there is a path $u, w_1, w_2, \dots, w_k, v$ in $G(V, E)$ connecting u and v .

(ii) We prove the forward implication by contradiction. If $B_0(V_0, I_0)$ is disconnected, then it can be divided into two parts. According to the evolving process, each time a new node i joins I , i will choose a prototype and connect to some nodes in V , which are already linked to the prototype. Thus, if the network is not connected when i joins, it will remain disconnected. Similar conclusion can be drawn when a new node joins V . Thus, if $B_0(V_0, I_0)$ is disconnected, $B(V, I)$ should be disconnected either, a contradiction.

Conversely, similar to the argument about the forward implication, the advent of new users will not change the

connectivity of the initial bipartite graph. Thus, $B(V, I)$ will be connected if $B_0(V_0, I_0)$ is connected.

Combining cases (i) and (ii), we complete the proof.

B. Proof of Lemma 3

We first derive the probability that two users $v \in V$ and $i \in I$ are connected in $B(V, I)$. There are only two cases: v and i are connected by an edge or not connected. Let m denote the number of edges in $B(V, I)$. For the former case, it contains $m C_{m-1}^{d^B(i)-1} C_{m-d^B(i)}^{d^B(v)-1}$ events. The latter case contains $C_m^{d^B(i)} C_{m-d^B(i)}^{d^B(v)}$ events. Since a user selects each edge with equal probability in the generation process, the probability that v and i are connected could be derived according to the classic probability model

$$P(v, i) = \frac{m C_{m-1}^{d^B(i)-1} C_{m-d^B(i)}^{d^B(v)-1}}{m C_{m-1}^{d^B(i)-1} C_{m-d^B(i)}^{d^B(v)-1} + C_m^{d^B(i)} C_{m-d^B(i)}^{d^B(v)}}. \quad (1)$$

After simple calculation, we have

$$P(v, i) = \frac{d^B(v) d^B(i)}{m + d^B(v) d^B(i) - d^B(v) - d^B(i) + 1}. \quad (2)$$

Since both $d^B(v)$ and $d^B(i)$ is smaller than $n^{\frac{1}{4}}$, the denominator is $m(1 + o(1))$. According to the Khinchin's law, when $n \rightarrow \infty$, m could be expressed as $(1 \pm o(1))n\bar{d}$ with probability 1. Then the probability that $v \in V$ and $i \in I$ are connected is

$$P(v, i) = \frac{d^B(v) d^B(i)}{n\bar{d}}. \quad (3)$$

In a similar way, the probability that user $w \in V$ connects to user $i \in I$ is

$$P(w, i) = \frac{d^B(w) d^B(i)}{n\bar{d}}. \quad (4)$$

Then, the probability that v and w are connected in $G(V, E)$ because of the common neighbor i is

$$P(v, w|i) = \frac{d^B(v) d^B(w) [d^B(i)]^2}{n^2 \bar{d}^2}. \quad (5)$$

The probability that v and w are connected in $G(V, E)$ is

$$\begin{aligned}
P(v, w) &= 1 - \prod_{i \in I} \left(1 - \frac{d^B(v)d^B(w) [d^B(i)]^2}{n^2 \bar{d}^2} \right) \\
&= 1 - e^{-\sum_{i \in I} \ln \left(1 - \frac{d^B(v)d^B(w) [d^B(i)]^2}{n^2 \bar{d}^2} \right)} \\
&= 1 - e^{-\sum_{i \in I} \left(-\frac{d^B(v)d^B(w) [d^B(i)]^2}{n^2 \bar{d}^2} \right)} \\
&= \frac{d^B(v)d^B(w) \text{vol}_2^B(I)}{n^2 \bar{d}^2}.
\end{aligned} \tag{6}$$

To derive the last two equations, we need to verify that $\frac{d^B(v)d^B(w) \text{vol}_2^B(I)}{n^2 \bar{d}^2}$ is an infinitesimal. First, we notice that the expected degree \bar{d} is a constant. Since the degree sequence of V in $B(V, I)$ follows a power law with exponent $2 + 1/\tau$, the expected degree is

$$\bar{d} = \sum_{k=1}^{n^\gamma} k \frac{C}{k^{2+1/\tau}} = \Theta(1).$$

Furthermore, since $d^B(v) < n^\gamma, d^B(w) < n^\gamma, \text{vol}_2^B(I) = \Theta(n^{1+\gamma(1-\tau)})$ and $\gamma < \frac{1}{4+\tau}$, we derive that $\frac{d^B(v)d^B(w) \text{vol}_2^B(I)}{n^2 \bar{d}^2}$ is $o(1)$. Because $\ln(1-x)$ and $-x, 1 - e^{-x}$ and $-x$ are two pairs of equivalent infinitesimals when $x \rightarrow 0$, we obtain the last two equations in 6 when $n \rightarrow \infty$.

C. Proof of Lemma 4

Let X_j be the indicator variable that user $j \in T$ is in $N(S)$. $X_j = 1$ when j connects to at least one node in S , thus,

$$P(X_j = 1) = 1 - \prod_{i \in S} \left(1 - \frac{d^B(i)d^B(j) \text{vol}_2^B(I)}{n^2 \bar{d}^2} \right). \tag{7}$$

Let $\rho = \frac{\text{vol}_2^B(I)}{n^2 \bar{d}^2}$, by Section I-G we prove the following inequality

$$P(X_j = 1) \geq \text{vol}^B(S) d^B(j) \rho - [\text{vol}^B(S) d^B(j) \rho]^2. \tag{8}$$

According to the relationship between $d^G(u)$ and $d^B(u)$ revealed in Lemma 2, we have the following inequality in expectation,

$$P(X_j = 1) \geq \frac{\text{vol}^G(S) d^G(j) \rho}{n^{2\gamma(1-\tau)}} - \frac{[\text{vol}^G(S) d^G(j) \rho]^2}{n^{4\gamma(1-\tau)}}. \tag{9}$$

According to the definition of $\text{vol}^G(\cdot)$ and X_j , we have $\text{vol}^G(N(S) \cap T) = \sum_{j \in T} d^G(j) X_j$. The expectation of $\text{vol}^G(N(S) \cap T)$ is greater than

$$\begin{aligned}
&\frac{\text{vol}^G(S) \rho \sum_{j \in T} (d^G(j))^2}{n^{2\gamma(1-\tau)}} - \frac{[\text{vol}^G(S) \rho]^2 \sum_{j \in T} (d^G(j))^3}{n^{4\gamma(1-\tau)}} \\
&= \frac{\text{vol}^G(S) \rho \text{vol}_2^G(T)}{n^{2\gamma(1-\tau)}} - \frac{[\text{vol}^G(S) \rho]^2 \text{vol}_3^G(T)}{n^{4\gamma(1-\tau)}}.
\end{aligned} \tag{10}$$

Applying Lemma 5.1 in [A1], with probability $1 - e^{-c}$, the following formula holds

$$\text{vol}^G(N(S) \cap T) = \sum_{j \in T} d^G(j) X_j$$

$$\begin{aligned}
&\geq \frac{\text{vol}^G(S) \rho \text{vol}_2^G(T)}{n^{2\gamma(1-\tau)}} - \frac{[\text{vol}^G(S) \rho]^2 \text{vol}_3^G(T)}{n^{4\gamma(1-\tau)}} \\
&\quad - \sqrt{\frac{2c \text{vol}^G(S) \text{vol}_3^G(T) \rho}{n^{2\gamma(1-\tau)}}}.
\end{aligned}$$

Let the last two terms be no greater than

$$\frac{\varepsilon \text{vol}^G(S) \rho \text{vol}_2^G(T)}{n^{2\gamma(1-\tau)}},$$

we obtain the two conditions and the conclusion that

$$\text{vol}^G(N(S) \cap T) \geq (1 - 2\varepsilon) \frac{\text{vol}_2^G(T) \text{vol}^G(S) \text{vol}_2^B(I)}{n^{2+2\gamma(1-\tau)} \bar{d}^2}. \tag{11}$$

D. Proof of Lemma 6

Let $X_i = 1$ denote that the i -th user falls in this region and $X = \sum_{i=1}^n X_i$ denote the number of users in the region. It is obvious that $P(X_i = 1) = A/n$. According to Lemma 2.1 in [34], by Chernoff bound, for any constant $\delta \in (0, 1)$, we have

$$P(|X - E(X)| \geq \delta E(X)) \leq 2e^{-E(X)\delta^2/3},$$

where $E(X) = \sum_{i=1}^n E(X_i) = A$. Again, $A = \omega(\log n)$ and δ is a constant. Thus, $(1 - \delta)A \leq X \leq (1 + \delta)A$ and $X = \Theta(A)$ with probability $1 - o(1)$, i.e., the number of users falling in this region is bounded by $\Theta(A)$.

E. Proof of Lemma 7

Randomly index the K users by number $1, 2, \dots, K$. Let X_1, X_2, \dots, X_K denote the the time when the users obtain the content respectively, which are independently and identically distributed. The probability that $X_i = T$ is $(1 - p)^{T-1} p$, i.e., X_i follows a geometric distribution with parameter p . Let X denote the time when the K users obtain the content, it is evident that $X = \max_{i=1, \dots, K} X_i$. Let $q = 1 - p, q \in (0, 1)$, we next derive the distribution function of X .

$$\begin{aligned}
P(X < T) &= P(X_1 < T, \dots, X_K < T) = \prod_{j=1}^K P(X_j < T) \\
&= \prod_{j=1}^K \left[\sum_{m=1}^{T-1} p q^{m-1} \right] = \frac{p}{q} \prod_{j=1}^K \left[\sum_{m=1}^{T-1} q^m \right] \\
&= \frac{p}{q} \prod_{j=1}^K \frac{q(1 - q^{T-1})}{1 - q} = (1 - q^{T-1})^K.
\end{aligned} \tag{12}$$

Based on the distribution, we further deduce the expectation. Since X is a non-negative integer-valued random variable, we have

$$\begin{aligned}
E(X) &= \sum_{T=1}^{\infty} P(X \geq T) = \sum_{T=1}^{\infty} [1 - P(X < T)] \\
&= \sum_{T=1}^{\infty} [1 - (1 - q^{T-1})^K] = \sum_{T=1}^{\infty} \left[1 - \sum_{j=0}^{K-1} C_K^j 1^{K-j} (-q^{T-1})^j \right] \\
&= \sum_{T=1}^{\infty} \left[1 - \left[\sum_{j=1}^K C_K^j 1^{K-j} (-q^{T-1})^j + 1 \right] \right]
\end{aligned}$$

$$\begin{aligned}
&= -\sum_{T=1}^{\infty} \sum_{j=1}^K C_K^j (-q^{T-1})^j = -\sum_{j=1}^K C_K^j (-1)^j \sum_{T=1}^{\infty} (q^j)^{T-1} \\
&= -\sum_{j=1}^K C_K^j (-1)^j \frac{1}{1-q^j} = -\sum_{j=1}^K C_K^j (-1)^j \frac{1}{1-(1-p)^j}.
\end{aligned}$$

Since $p \rightarrow 0$ when $n \rightarrow \infty$ and $(1-p)^j \approx 1-jp$, the expectation is equivalent to

$$E(X) = -\frac{1}{p} \sum_{j=1}^K \frac{C_K^j (-1)^j}{j}. \quad (13)$$

To derive $E(X)$, we focus on calculating $\sum_{j=1}^K \frac{C_K^j (-1)^j}{j}$. Let $a_K = \sum_{j=1}^K \frac{C_K^j (-1)^j}{j}$, then

$$\begin{aligned}
a_{K+1} &= \sum_{j=1}^{K+1} \frac{C_{K+1}^j (-1)^j}{j} = \sum_{j=1}^K \frac{C_{K+1}^j (-1)^j}{j} + \frac{(-1)^{K+1}}{K+1}, \\
a_{K+1} - a_K &= \sum_{j=1}^K \frac{(-1)^j}{j} (C_{K+1}^j - C_K^j) + \frac{(-1)^{K+1}}{K+1} \\
&\stackrel{(1)}{=} \frac{1}{K+1} \sum_{j=1}^K (-1)^j C_{K+1}^j + \frac{(-1)^{K+1}}{K+1} \\
&= \frac{1}{K+1} \left[\sum_{j=1}^{K+1} (-1)^j C_{K+1}^j - (-1)^{K+1} \right] + \frac{(-1)^{K+1}}{K+1} \\
&= \frac{1}{K+1} \left[\sum_{j=0}^{K+1} (-1)^j C_{K+1}^j - 1 - (-1)^{K+1} \right] + \frac{(-1)^{K+1}}{K+1} \\
&= \frac{1}{K+1} [0 - 1 - (-1)^{K+1}] + \frac{(-1)^{K+1}}{K+1} = \frac{-1}{K+1},
\end{aligned}$$

where equation (1) is due to $C_{K+1}^j = C_K^j + C_K^{j-1}$ and $\frac{1}{j} C_K^{j-1} = \frac{1}{K+1} C_{K+1}^j$.

Since $a_1 = -1$, we have $a_K = -\sum_{j=1}^K \frac{1}{j}$. Due to the harmonic series, $a_K = -\Theta(\log K)$ and thus

$$E(X) = \Theta\left(\frac{\log K}{p}\right). \quad (14)$$

Markov inequality states that for a non-negative random variable X and $t > 0$, $P(X \geq t) \leq E(X)/t$. Here, we let $t = E(X) \log n$, and obtain

$$X = O\left(\frac{\log K \log n}{p}\right),$$

with probability $1 - \frac{1}{\log n}$. This completes the proof.

F. Proof of Theorem 4

We divide the transmission into two stages: (i) the transmission in $V \setminus S_{\lceil i/2 \rceil}(s)$, (ii) the transmission in $S_{\lceil i/2 \rceil}(s)$, and derive the transmission time T_a and T_b respectively. Then we can obtain T_D by summing T_a and T_b .

(i) We first consider the transmission in $V \setminus S_{\lceil i/2 \rceil}(s)$. To obtain an upper bound, we calculate the time needed to satisfy each tier of users $N_h(s)$ ($\lceil i/2 \rceil < h \leq D$) and summarize the

time as the final result. And we assume users in $N_h(s)$ begin to receive the content when users socially closer to the source already have the content.

For some eager user $u \in N_h(s)$, referring to the argument in Lemma 5, its social distance to the source is denoted as $h > \lceil i/2 \rceil$. There must exist a user w who is $\lceil i/2 \rceil$ hops away from u and $h - \lceil i/2 \rceil$ hops away from the source. Then, the users in $S_{\lceil i/2 \rceil}(w)$ are exactly within i hops of u . At each time slot, the probability that there is at least one user in $S_{\lceil i/2 \rceil}(w)$ within geographic range is

$$p_1 = 1 - \left(1 - \frac{\pi L_r^2}{4n}\right)^{|S_{\lceil i/2 \rceil}(w)|} = \frac{\pi L_r^2 |S_{\lceil i/2 \rceil}(w)|}{4n}. \quad (15)$$

Thus, the quantity of users who meet users within i and L_r is $\pi L_r^2 |S_{\lceil i/2 \rceil}(w)| |N_h(s)| / 4n$, which upperly bounds the transmitters at the time slot. The probability that user u is not interfered, i.e., no transmitter is within $(1 + \Delta)L_r$, is greater than

$$p_2 = \left(1 - \frac{\pi(1 + \Delta)^2 L_r^2}{4n}\right)^{p_1 |N_h(s)|}. \quad (16)$$

Then, the probability that u could find a holder and receive the content is

$$\begin{aligned}
p &= p_1 p_2 = \frac{\pi L_r^2 |S_{\lceil i/2 \rceil}(w)|}{4n} \left(1 - \frac{\pi(1 + \Delta)^2 L_r^2}{4n}\right)^{p_1 |N_h(s)|} \\
&\stackrel{(a)}{=} \frac{\pi L_r^2 |S_{\lceil i/2 \rceil}(w)|}{4n} \left(1 - \frac{\pi(1 + \Delta)^2 L_r^2}{4n} p_1 |N_h(s)|\right) \\
&= \frac{\pi L_r^2 |S_{\lceil i/2 \rceil}(w)|}{4n} \left(1 - \frac{\pi^2 (1 + \Delta)^2 L_r^4 |S_{\lceil i/2 \rceil}(w)| |N_h(s)|}{16n^2}\right),
\end{aligned} \quad (17)$$

where equality (a) is due to $(1 - x)^\alpha \approx 1 - \alpha x$ for $x \rightarrow 0$. Replace L_r with $\sqrt{n/\pi(1 + \Delta)} \sqrt{|S_{\lceil i/2 \rceil}(w)| |N_h(s)| \log n}$ in Equation 17, we see that the probability of interference is $1/16 \log n$. Thus, we have

$$p = \frac{\pi L_r^2 |S_{\lceil i/2 \rceil}(w)|}{4n} \left(1 - \frac{1}{16 \log n}\right). \quad (18)$$

Replace the probability p in Lemma 6, we obtain the time slots needed to satisfy all the users in $N_h(s)$ with probability $1 - o(1)$.

$$\begin{aligned}
T_h &\leq \frac{4n \log(|N_h(s)|) \log n}{(1 - o(1)) \pi L_r^2 |S_{\lceil i/2 \rceil}(w)|} \\
&\stackrel{(b)}{=} \frac{4n \log(|N_h(s)|) \log n \pi(1 + \Delta) \sqrt{|S_{\lceil i/2 \rceil}(w)| |N_h(s)| \log n}}{\pi |S_{\lceil i/2 \rceil}(w)| n} \\
&= 4(1 + \Delta) \log(|N_h(s)|) \log n \sqrt{\frac{|N_h(s)| \log n}{|S_{\lceil i/2 \rceil}(w)|}} \\
&\stackrel{(c)}{<} 4(1 + \Delta) \log^{2.5} n \sqrt{|N_h(s)| / \mu n^{\gamma(\lceil i/2 \rceil - \tau)}}, \quad (19)
\end{aligned}$$

where equality (b) is obtained by substituting L_r , and inequality (c) is derived by replacing $|S_{\lceil i/2 \rceil}(w)|$ with its lower bound.

Then, we are ready to derive the transmission time in

$V \setminus S_{\lceil i/2 \rceil}(s)$.

$$\begin{aligned}
T_a &= \sum_{h=\lceil i/2 \rceil+1}^D T_h < \frac{4(1+\Delta) \log^{2.5} n}{\sqrt{\mu n^{\gamma(\lceil i/2 \rceil - \tau)}}} \sum_{h=\lceil i/2 \rceil+1}^D \sqrt{|N_h(s)|} \\
&< \frac{4(1+\Delta) \log^{2.5} n}{\sqrt{\mu n^{\gamma(\lceil i/2 \rceil - \tau)}}} \sum_{h=0}^D \sqrt{|N_h(s)|} \\
&\stackrel{(d)}{\leq} \frac{4(1+\Delta) \log^{2.5} n}{\sqrt{\mu n^{\gamma(\lceil i/2 \rceil - \tau)}}} \sqrt{nD} \\
&= 4(1+\Delta) \log^{2.5} n \sqrt{n^{1-\gamma(\lceil i/2 \rceil - \tau)} D / \mu}, \quad (20)
\end{aligned}$$

where inequality (d) is due to a simple transformation from the fact that the arithmetic mean of a set of positive numbers is less than or equal to the quadratic mean, i.e., $b_1 + b_2 + \dots + b_m \leq \sqrt{m} \sqrt{b_1^2 + b_2^2 + \dots + b_m^2}$.

(ii) We next examine the transmission time in $S_{\lceil i/2 \rceil}(s)$. To derive an upper bound, we assume all the users in $S_{\lceil i/2 \rceil}(s)$ only request the content from the source. Similar to the idea in (i), we have the probability p_1

$$p_1 = \frac{\pi L_c^2}{4n} \quad (21)$$

and probability p_2

$$p_2 = \left(1 - \frac{\pi(1+\Delta)^2 L_c^2}{4n} p_1 |S_{\lceil i/2 \rceil}(s)|\right). \quad (22)$$

Accordingly, the probability that a user in $S_{\lceil i/2 \rceil}(s)$ could obtain the content is $p = p_1 p_2$

$$\begin{aligned}
p &= p_1 p_2 = \frac{\pi L_c^2}{4n} \left(1 - \frac{\pi^2(1+\Delta)^2 L_c^4}{16n^2} |S_{\lceil i/2 \rceil}(s)|\right) \\
&= \frac{\pi L_c^2}{4n} \left(1 - \frac{1}{16 \log n}\right) \\
&= \frac{1}{4(1+\Delta) \sqrt{|S_{\lceil i/2 \rceil}(s)| \log n}} (1 - o(1)), \quad (23)
\end{aligned}$$

where the probability of interference is $o(1)$ with $L_c = \sqrt{n/\pi(1+\Delta)} \sqrt{|S_{\lceil i/2 \rceil}(s)| \log n}$. Applying Lemma 7, T_b is bounded by

$$\begin{aligned}
T_b &\leq 4(1+\Delta) \log^{1.5} n \log |S_{\lceil i/2 \rceil}(s)| \sqrt{|S_{\lceil i/2 \rceil}(s)|} \\
&< 4(1+\Delta) \sqrt{n^{\lceil i/2 \rceil/D}} \log^{2.5} n. \quad (24)
\end{aligned}$$

In fact, T_b is in a smaller scale than T_a . When $i = 1$, it is clear that $T_b = o(T_a)$. When $i \in [2, D-1]$, we have $D - \lceil i/2 \rceil + \tau > \lceil i/2 \rceil$. Since $\gamma < 1/D$, we have $D - \gamma D(\lceil i/2 \rceil - \tau) > D - \lceil i/2 \rceil + \tau$, i.e., $1 - \gamma(\lceil i/2 \rceil - \tau) > \lceil i/2 \rceil/D$, and accordingly $n^{\lceil i/2 \rceil/D} = o(n^{1-\gamma(\lceil i/2 \rceil - \tau)})$. Since μ is a constant, $T_b = o(T_a)$. Thus, the transmission time over the whole network $T_D = T_a + T_b$ is upper bounded by $O\left(4(1+\Delta) \log^{2.5} n \sqrt{n^{1-\gamma(\lceil i/2 \rceil - \tau)} D / \mu}\right)$. This completes the proof.

G. Proof of Inequality 8

Since the product is only about i , for simplicity, we could rewrite the inequality by denoting $\frac{d^B(i) d^B(j) \text{vol}_2^B(I)}{n^2 \bar{d}^2}$ as $a_i \in$

$(0, 1)$. Then, it is equivalent to proving

$$1 - \prod_{i=1}^{|S|} (1 - a_i) \geq \sum_{i=1}^{|S|} a_i - \left(\sum_{i=1}^{|S|} a_i\right)^2. \quad (25)$$

We prove this inequality by mathematical induction.

(i) We first verify the basic case. When $i = 1$, the left hand side is a_1 and the right hand side is $a_1 - a_1^2$. Inequality 25 holds.

When $i = 2$, the left hand side is $a_1 + a_2 - a_1 a_2$ and the right hand side is $a_1 + a_2 - a_1^2 - a_2^2 - 2a_1 a_2$. Inequality 25 is equivalent to $a_1^2 + a_2^2 + a_1 a_2 \geq 0$. Since $0 < a_i < 1$, the inequality holds.

(ii) We assume Inequality 25 holds when $|S| = k$, i.e., $1 - \prod_{i=1}^k (1 - a_i) \geq \sum_{i=1}^k a_i - \left(\sum_{i=1}^k a_i\right)^2$. When $|S| = k+1$, Inequality 25 becomes

$$\begin{aligned}
1 - \prod_{i=1}^{k+1} (1 - a_i) &\geq \sum_{i=1}^{k+1} a_i - \left(\sum_{i=1}^{k+1} a_i\right)^2 \\
\Leftrightarrow 1 - (1 - a_{k+1}) \prod_{i=1}^k (1 - a_i) &\geq \sum_{i=1}^k a_i + a_{k+1} - \left(\sum_{i=1}^k a_i + a_{k+1}\right)^2 \\
\Leftrightarrow 1 - \prod_{i=1}^k (1 - a_i) + a_{k+1} \prod_{i=1}^k (1 - a_i) &\geq \\
\sum_{i=1}^k a_i + a_{k+1} - \left(\sum_{i=1}^k a_i\right)^2 - a_{k+1}^2 - 2a_{k+1} \sum_{i=1}^k a_i. &
\end{aligned}$$

Since $1 - \prod_{i=1}^k (1 - a_i) \geq \sum_{i=1}^k a_i - \left(\sum_{i=1}^k a_i\right)^2$, we then aim to prove

$$\begin{aligned}
\sum_{i=1}^k a_i - \left(\sum_{i=1}^k a_i\right)^2 + a_{k+1} \prod_{i=1}^k (1 - a_i) &\geq \\
\sum_{i=1}^k a_i + a_{k+1} - \left(\sum_{i=1}^k a_i\right)^2 - a_{k+1}^2 - 2a_{k+1} \sum_{i=1}^k a_i & \\
\Leftrightarrow a_{k+1} \prod_{i=1}^k (1 - a_i) &\geq a_{k+1} - a_{k+1}^2 - 2a_{k+1} \sum_{i=1}^k a_i.
\end{aligned}$$

Since $0 < a_i < 1$, we divide a_{k+1} in both sides and obtain

$$\prod_{i=1}^k (1 - a_i) \geq 1 - a_{k+1} - 2 \sum_{i=1}^k a_i.$$

To prove this inequality, we apply the Bernoulli inequality $\prod_{i=1}^k (1 - a_i) \geq 1 - \sum_{i=1}^k a_i$. Then, it is equivalent to proving

$$1 - \sum_{i=1}^k a_i \geq 1 - a_{k+1} - 2 \sum_{i=1}^k a_i,$$

which is clearly true since $0 < a_i < 1$. Thus, Inequality 25 holds when $|S| = k+1$.

Summarizing (i) and (ii), we could reach the conclusion that Inequality (25) holds.

II. DISCUSSIONS

A. Discussion on Variable Physical Density

In this section, we discuss the case where the network area will not extend to accommodate new users, i.e., the physical density keeps increasing as the network evolves.

1) *Settings and Assumptions:* Before looking into this case, we first articulate the basic settings and necessary assumptions. Users are independently and uniformly distributed on a square of width a , where a is a constant. We assume that the physical size of users tends to zero, otherwise the network could only hold constant number of users and we cannot obtain asymptotic results. Since only the setting of physical layer is changed, results of the social layer still hold, of which Theorem 1 is needed, i.e., the size of neighborhood in evolving networks.

2) *Transmission Scheme:* Under a different setting, the transmission scheme accordingly needs some modification. Specifically, the geographic range should be redesigned and the physical transmission strategy needs to be modified. We first illustrate how to construct the highway system in a fixed network area. In fact, the construction technique is also given in [34]. The network area is still divided into small squares in the same way as before, except that the length is rescaled to be $\frac{ac}{\sqrt{n}}$, where c is the same constant as previous. And the authors in [34] confirm that the properties still hold i.e., the highways are almost straight lines; the transmission rate between two relays in the highway is constant R ; and each relay only serves nodes within a slab of width $\omega' = \frac{\omega a}{\sqrt{n}}$ (note that the width is rescaled).

3) *Derivation of Transmission Time:* Following the idea of the main paper, we need to first figure out a proper setting of geographic range L . The geographic range should guarantee that, given social depth i , any user should find at least one neighbor with both i social hops and L geographic distance. We set L with the same idea of Lemma 5. Consider a user w with m neighbors, let X_k be the indicator variable that the distance between w and the k -th user is within L . Note that the physical density becomes $\frac{n}{a^2}$ in the new setting. Similar to the proof of Lemma 5, we can see that the probability of $X_k = 1$ is no less than $\frac{\pi m L^2}{4a^2}$. Then, if we set $L = 8\sqrt{a^2 \log n / \pi m}$ by Lemma 2.1 in [35], there are at least $8 \log n$ users within L among the m users with probability $1 - o\left(\frac{1}{n}\right)$. The rest derivation is the same as Lemma 5 where we substitute m with the lower bound of $|S_{\lfloor \frac{i}{2} \rfloor}(s)|$ which is $\mu n^{\gamma(\lfloor i/2 \rfloor - \tau)}$. Thus, we have $L = 8\sqrt{a^2 \log n / \pi \mu n^{\gamma(\lfloor i/2 \rfloor - \tau)}}$.

Next, we also analyze the bit rate of each transmission from the perspective of the number of transmissions each relay has to serve. In the highway system, each relay only serves users in a slab of area $2L \times \omega'$. By Chernoff bound, the number of users fall in this area is $O(L\omega'n/a^2) = O(L\omega\sqrt{n}/a)$, i.e., each relay serves at most $O(L\omega\sqrt{n}/a)$ transmissions. Recall that, the transmission rate of transceivers in the highway system is constant R . Then, the bit rate of each transmission is $\Omega(Ra/L\omega\sqrt{n})$, where ω is a constant defined in the highway system. We could further infer that each transmission takes at most $cL\sqrt{n}/Ra$ time slots to complete.

With the above analysis, we are ready to derive the transmission time. We attempt to prove a recursion similar to the main paper, i.e., $T_k \leq kc \log n L F \sqrt{n}/Ra$. We also prove the inequality by induction. The proof is nearly the same as the main paper, except that the upper bound of time slots taken by each transmission becomes $cL\sqrt{n}/Ra$. Thus, we omit the detailed derivation for brevity, and the readers may substitute $cLFR$ with $cL\sqrt{n}/Ra$ to obtain the whole proof. Finally, let $k = D$ and $L = 8\sqrt{a^2 \log n / \pi \mu n^{\gamma(\lfloor i/2 \rfloor - \tau)}}$, we obtain that

$$T_D = O\left(DF\sqrt{n^{1-\gamma(\lfloor i/2 \rfloor - \tau)}} \log^{\frac{3}{2}} n/R\right),$$

which is the same as the main paper. Similar technique could be applied to the lower bound and the mobile case, and we omit the details for brevity.

Intuitively, although the network area becomes a constant, we could rescale the geographic range L to adapt to this setting to avoid possible interferences and reach the same performance. Specifically, the setting of fixed network area brings about the disadvantage of severer interference. However, accompanying with that, the advantage is that users are easier to find a neighbor within reach. We keep the standard of setting L unchanged, i.e., a user could find at least $8 \log n$ users that are within i social hops and L physical distance with probability $1 - \frac{1}{n}$. The interference and accessibility of legitimate neighbors will compromise. And the performance will not be hurt.

B. Discussion on Probabilistic Formation of $G(V, E)$

In the main paper, the Affiliation Network model is adopted to generate the social connections, where we obtain the social network $G(V, E)$ from the bipartite graph $B(V, I)$. Specifically, in the model, any two users (e.g., $u, v \in V$) that belong to the same affiliation (e.g., $i \in I$) in $B(V, I)$ are socially connected. Nevertheless, in practice, this may not always be true. Thus, in this part, we would like to discuss the case where users in the same affiliation are probabilistically connected.

Firstly, let us define the probabilistic formation process. To put it formally, the probability that u and v are connected is set to be $(d^B(i))^\alpha$, where $\alpha \in (0, 1)$ is a constant. Under this setting, we could apply some results given in Section 8 of [11]: (1) The diameter of $G(V, E)$ is still constant. (2) The network $G(V, E)$ is still densifying over time. (3) The degree distribution of $G(V, E)$ is still heavy-tailed. Since users in the same affiliation are not definitely connected, Lemma 1 will not hold and an additional assumption is also needed that the social network is connected (if not connected, we focus on the largest component), otherwise the transmission time is infinite.

For ease of understanding our analytical process, we further make two statements. (1) Changing the formation of social network $G(V, E)$ has evident impact on the analysis of social properties but negligible impact on the derivation of transmission time, since it only applies the final results of social properties (i.e., the size of neighborhood). (2) The model of the bipartite graph $B(V, I)$ is not changed. Thus, the properties of $B(V, I)$ still hold.

By far, we could reap some insights into this problem. Let us still denote the diameter as D and characterize the degree dis-

tribution of $G(V, E)$ by $1 + \theta$. Since the network still densifies, we can infer that $\theta < 1$. To explain, if $\theta \geq 1$, the total number of edges is $\frac{1}{2} \times \sum_{k=\log^{2+\epsilon} n}^{n^\gamma} nP(\text{the degree of a user is } k) \cdot k = \frac{1}{2} \times \sum_{k=\log^{2+\epsilon} n}^{n^\gamma} n \frac{C}{k^{1+\theta}} k = \frac{1}{2} \times \sum_{k=\log^{2+\epsilon} n}^{n^\gamma} n \frac{C}{k^\theta} = \tilde{\Theta}(n)$ (the notation $\tilde{\Theta}$ omits possible log terms), which is in almost the same scale of users, contradicting with the densification law.

We next come to modify the previous results about the social network. Regarding Lemma 2, Following the proof of Lemma 2, we see that if an edge of user $u \in V$ in $B(V, I)$ points to a node $i \in I$, the edge brings the neighbors brought by the edge is no longer $Y_i - 1$ but $\frac{Y_i - 1}{Y_i^\alpha}$. Then, the degree of u in $G(V, E)$ should be $d^G(u) = \sum_{i=1}^{d^B(u)} \frac{Y_i - 1}{Y_i^\alpha}$. Resultantly, the relationship between $d^G(u)$ and $d^B(u)$ becomes $E[d^G(u)] = E[d^B(u)] \Theta(n^{\gamma(1-\tau-\alpha)})$. Recall that the degree of user u in $B(V, I)$ follows a power-law distribution of exponent $2 + \frac{1}{\tau} > 2$. Then, $E[d^B(u)]$ is a constant and $E[d^G(u)] = \Theta(n^{\gamma(1-\tau-\alpha)})$. Moreover, $E[d^G(u)] = \sum_{k=\log^{2+\epsilon} n}^{n^\gamma} k \cdot \frac{C}{k^{1+\theta}} = \Theta(n^{\gamma(1-\theta)})$. Thus, we have

$$\theta = \tau + \alpha. \quad (26)$$

Then, we proceed to Lemma 3. The derivation of $P(v, i)$ and $P(w, i)$ still holds, since it is based on $B(V, I)$. However, the probability that v and w are connected in $G(V, E)$ because of the common neighbor i becomes

$$P(v, w|i) = \frac{d^B(v)d^B(w) [d^B(i)]^2}{n^2 \bar{d}^2} \cdot \frac{1}{[d^B(i)]^\alpha},$$

where the term $\frac{1}{[d^B(i)]^\alpha}$ is imposed, since under the new setting two users of the same affiliation i are connected with probability $[d^B(i)]^{-\alpha}$. Consequently, the probability that v and w are connected in $G(V, E)$ is

$$P(v, w) = \frac{d^B(v)d^B(w) \text{vol}_{2-\alpha}^B(I)}{n^2 \bar{d}^2}.$$

In terms of the key lemma, i.e., Lemma 4 in the main paper. We will show how to obtain two new inequalities via minor modification of the proof. The proof of the new Lemma 4 is almost the same as before, except that the value of ρ becomes $\frac{\text{vol}_{2-\alpha}^B(I)}{n^2 \bar{d}^2}$. After applying the modified Lemma 2, the inequality about $P(X_j = 1)$ becomes

$$P(X_j = 1) \geq \frac{\text{vol}^G(S) d^G(j) \rho}{n^{2\gamma(1-\tau-\alpha)}} - \frac{[\text{vol}^G(S) d^G(j) \rho]^2}{n^{4\gamma(1-\tau-\alpha)}}.$$

Following the same technique of Lemma 4, we have the new version of Lemma 4. To put it formally, we state the new lemma as follows.

For the social network $G(V, E)$, given two subsets $S \subseteq V, T \subseteq V$ and $S \cap T = \emptyset$, if

$$\frac{2c \text{vol}_3^G(T)}{\varepsilon^2 (\text{vol}_2^G(T))^2} \frac{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2}{\text{vol}_{2-\alpha}^B(I)} \leq \text{vol}^G(S), \quad (27)$$

$$\text{vol}^G(S) \leq \frac{\varepsilon \text{vol}_2^G(T)}{\text{vol}_3^G(T)} \frac{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2}{\text{vol}_{2-\alpha}^B(I)}, \quad (28)$$

then, with probability $1 - e^{-c}$ we have

$$\text{vol}^G(N(S) \cap T) \geq (1 - 2\varepsilon) \frac{\text{vol}_2^G(T) \text{vol}^G(S) \text{vol}_{2-\alpha}^B(I)}{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2}. \quad (29)$$

Now we come to the last phase of deriving the bound of neighborhood size. For the upper bound, the proof is the same as before and the result is still $|S_i(u)| \leq n^{\frac{1}{D}}$. Regarding the lower bound, we attempt to derive it by applying the new Lemma 4. The key is to verify the two inequalities.

For brevity, we only indicate the proof of the inequalities. For basic preparations, the readers may refer to the proof of Theorem 1. Here are some useful results under the new setting: $\text{vol}_2^G(T_i(u)) = \int_{\log^{2+\epsilon} n}^{n^\gamma} n \frac{C}{x^{1+\theta}} x^2 dx = (1 + o(1)) n^{1+\gamma(2-\theta)}$, $\text{vol}_3^G(T_i(u)) = \int_{\log^{2+\epsilon} n}^{n^\gamma} n \frac{C}{x^{1+\theta}} x^3 dx = (1 + o(1)) n^{1+\gamma(3-\theta)}$. And $\text{vol}_{2-\alpha}^B(I) = \int_1^{n^\gamma} n \frac{C}{x^{2+\tau}} x^{2-\alpha} dx = \int_1^{n^\gamma} n \frac{C}{x^{\alpha+\tau}} dx$. Moreover, recall that $\alpha + \tau = \theta < 1$, we have that $\text{vol}_{2-\alpha}^B(I) = \frac{n^{\gamma(1-\tau-\alpha)}}{n \bar{d}^2}$.

As for Inequality 27, we have

$$\begin{aligned} & \frac{2c \text{vol}_3^G(T)}{\varepsilon^2 (\text{vol}_2^G(T))^2} \frac{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2}{\text{vol}_{2-\alpha}^B(I)} \\ &= \frac{2c \bar{d}^2 n^{2+2\gamma(1-\tau-\alpha)} n n^{\gamma(3-\theta)}}{\varepsilon^2 n^2 n^{2\gamma(2-\theta)} n n^{\gamma(1-\tau-\alpha)}} = \frac{2c \bar{d}^2}{\varepsilon^2} n^{\gamma(\theta-\tau-\alpha)} = \Theta(\log n). \end{aligned}$$

The last equality is due to $\alpha + \tau = \theta$ (Equation 26). Since all users in $G(V, E)$ have a degree greater than $\log^{2+\epsilon} n$, we have $\text{vol}^G(N_i(u)) > \Theta(\log n)$. Thus, the first condition is validated.

Regarding Inequality 28, it follows that

$$\begin{aligned} & \frac{\varepsilon \text{vol}_2^G(T)}{\text{vol}_3^G(T)} \frac{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2}{\text{vol}_{2-\alpha}^B(I)} \\ &= \frac{\varepsilon \bar{d}^2 n^{2+2\gamma(1-\tau-\alpha)} n n^{\gamma(2-\theta)}}{n n^{\gamma(3-\theta)} n n^{\gamma(1-\tau-\alpha)}} = \varepsilon \bar{d}^2 n^{1-\gamma(\tau+\alpha)} = \varepsilon \bar{d}^2 n^{1-\gamma\theta}, \end{aligned}$$

where the last equality is due to $\alpha + \tau = \theta$ (Equation 26). Following similar arguments in the proof of Theorem 1, we know that $\text{vol}^G(N_i(u))$ is upper bounded by $n^{\frac{1}{D}} n^{\gamma(1-\theta)}$. Since $i \leq D - 1$ and $\gamma < \frac{1}{D}$, we see that $\text{vol}^G(N_i(u))$ is upper bounded by $\varepsilon \bar{d}^2 n^{1-\gamma\theta}$. Thus, the second condition is verified.

Since the conditions of the new lemma 4 are satisfied, we have that

$$\begin{aligned} \text{vol}^G(N_{i+1}(u)) &\geq (1 - 2\varepsilon) \frac{\text{vol}_2^G(T_i(u)) \text{vol}^G(N_i(u)) \text{vol}_{2-\alpha}^B(I)}{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2} \\ &= \frac{(1 - 2\varepsilon) \text{vol}^G(N_i(u)) n n^{\gamma(2-\theta)} \cdot n n^{\gamma(1-\tau-\alpha)}}{n^{2+2\gamma(1-\tau-\alpha)} \bar{d}^2} \\ &= (1 - 2\varepsilon) \text{vol}^G(N_i(u)) n^\gamma / \bar{d}^2, \end{aligned}$$

which coincides with previous results. Then, the arguments and results in the proof of Theorem 1 could be applied seamlessly. And the derivation of transmission time is also the same as before. We can see that the probabilistic connection does not hurt the neighborhood size and the transmission evidently.

C. Discussion on Probabilistic User Behavior

In the main paper, it is potentially assumed that the transition from inactive state to eager state is deterministic due to practical concerns. Consider the example of a large public

event such as a conference, exhibition and the Hijj ritual in [1], where people constantly advent and join the activity. It is often the case that the participants join the activity due to similar interests, e.g., they want to learn recent advances of machine learning. Then, lots of content about the event would interest them, e.g., the agenda of the conference, an interesting report. When a user learns about such content via his/her social applications, he/she would be eager to obtain it.

Despite the rationality discussed above, the case of probabilistic state transition is still worthy of investigation. To take account of the probabilistic behavior, we assume that an inactive user (e.g., u) becomes eager due to its social neighbor (e.g., v) with probability reciprocal to $(d^B(i))^\alpha$, where i is their common neighbor in $B(V, I)$, $d^B(i)$ is the degree of i in $B(V, I)$ and $\alpha \in (0, 1)$ is a constant. Note that according to the social network model, since u and v are social neighbors, i.e., connected by an edge in $G(V, E)$, they must have a common neighbor in $B(V, I)$. Next, we take an equivalent view to the probabilistic formulation. Under the probabilistic setting, each edge (u, v) is associated with a probability $p_{uv} \propto (d^B(i))^{-\alpha}$, which indicates whether u (resp. v) will become eager (denotes as 1) or not (denoted as 0) due to v (resp. u). For edges with state 0, since the edges will not take effect, it is equivalent to deleting them or they are simply not generated in the formation of $G(V, E)$. Thus, to obtain the new social network, we only need to slightly revise the generation process. Previously, users belong to the same affiliation will be connected deterministically. Now, this process becomes probabilistic, i.e., users belong to the same affiliation are connected with probability reciprocal to $(d^B(i))^\alpha$. Then, we only need to consider transmissions in this new social graph.

After the above transformation, the subsequent analysis is the same as Section II-B, where each edge of $G(V, E)$ is generated with probability reciprocal to $(d^B(i))^\alpha$. For brevity, we omit the details of the analysis here to avoid redundancy.

D. Discussion on Non-trivial User Reaction time

In the main paper, for convenience of analysis, we assumed that eager users will become eager to request the content as soon as one of its neighbor is active, i.e., users' reaction time is zero. Although this treatment conforms with practice of previous studies, e.g., [2], we would like to further discuss the case of non-negligible user reaction time.

Let random variable Q denote users' reaction time, which takes value in (a, b) . We focus on the upper bound of transmission time, since in this case of non-trivial reaction time, the total transmission time is prolonged and the previous lower bound will certainly hold. Following the proof of Theorem 1, we see that the recursive inequality now becomes $T_k \leq (k)c \log nLF/R + kb$. To prove this inequality, we apply the induction technique as well.

(i) When $k = 0$, it is easy to see that the inequality still holds.

When $k = 1$, T_1 is the time when all neighbors of the source s get the content. To obtain an upper bound, we assume that transmissions start after all neighbors of s become eager. Then,

the time it takes to wait all neighbors to become eager is at most b slots, since the reaction time of each neighbor is $Q < b$ and independent of each other. The subsequent analysis of transmission time is the same as before and we could obtain the result that $T_1 \leq c \log(n^\gamma)LF/R + b < c \log nLF/R + b$.

(ii) We assume that $T_{k-1} \leq (k-1)c \log nLF/R + (k-1)b$ holds. At time T_{k-1} , all the users in $S_{k-1}(s)$ have the content. And after at most b time slots, all users in $N_k(s)$ will become eager, since the reaction time for each user is upper bounded by b . The subsequent transmission time is at most $c \log nLF/R$ time slots by Theorem 1. Sum the two parts, we have $T_k \leq T_{k-1} + c \log nLF/R + b \leq kc \log nLF/R + kb$. Thus, the transmission time of the whole network is now $T_D = O\left(DF\sqrt{n^{1-\gamma}(\lfloor i/2 \rfloor^{-\gamma})} \log^{\frac{3}{2}} n/R\right) + bD$.

If b is a constant irrelevant to n (which is often the case), the newly added term bD could be omitted. As for the mobile case, the upper bound could be derived in a similar way, and we omit the details for brevity.

E. Comparison with Other Schemes

To further validate the effectiveness of our schemes, although experimental comparisons are made, we would like to compare the analytical results with previous work. We look into previous researches of wireless networks and find four studies on transmission whose analytical results are compatible with our settings (especially, we assume no physical aid, such as cache, MIMO, cooperation, relay in the mobile case). Considering they are focused on transmission rate, we need to take one or two more steps to derive the transmission time based on their results.

In the static case, we present the results of [19], [A2] and [34] for comparison. In [19], the transmission rate of wireless networks is proven to be $O(\frac{1}{\sqrt{n}})$ and $\Omega(\frac{1}{\sqrt{n \log n}})$ each transmission pair. And a transmission scheme is developed to achieve the rate based on Voronoi tessellation. Then, the time for each transmission pair to finish transmitting is $O(\sqrt{n \log n})$ and $\Omega(\sqrt{n})$, where there is an evident gap from our results which are like $\sqrt{n^{1-\frac{1}{2}i}(\log n)^{\frac{3}{2}}}$. Another scheme is proposed in [A2], which achieves the same transmission rate and transmission time. Regarding [34], its scheme only improves the transmission lower bound to be $\Omega(\frac{1}{\sqrt{n}})$, i.e., the transmission time is $\Theta(\sqrt{n})$, which is still not comparable with ours.

In the mobile case, David Tse [20] took the head in analyzing the transmission rate under mobility model and found that under our setting of no relay, the transmission rate is just the same as the static case. That is to say, each transmission still needs $O(\frac{1}{\sqrt{n}})$ slots to finish. Although relay is allowed in [A2], its scheme only allows users to send data with rate $\Theta(\frac{1}{\sqrt{n(\log n)^3}})$. The resultant transmission time is $\Theta(\sqrt{n(\log n)^3})$, which still scales like \sqrt{n} and larger than ours ($\sqrt{n^{1-\frac{1}{2}i}(\log n)^{\frac{3}{2}}}$). Also with relays, the transmission scheme designed in [A3] significantly improves the transmission rate to be $\Theta(\frac{1}{(n\sqrt{\log n})^3})$. Then, the transmission time scales as

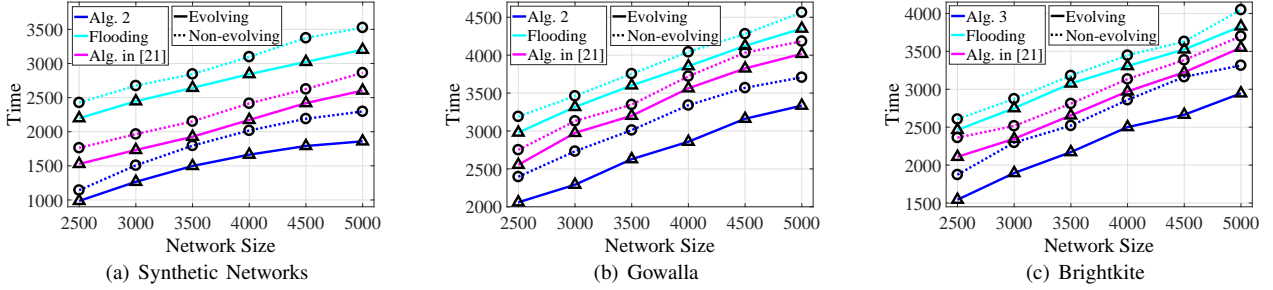


Fig. S1. Comparison of Transmission Time in Static Networks.

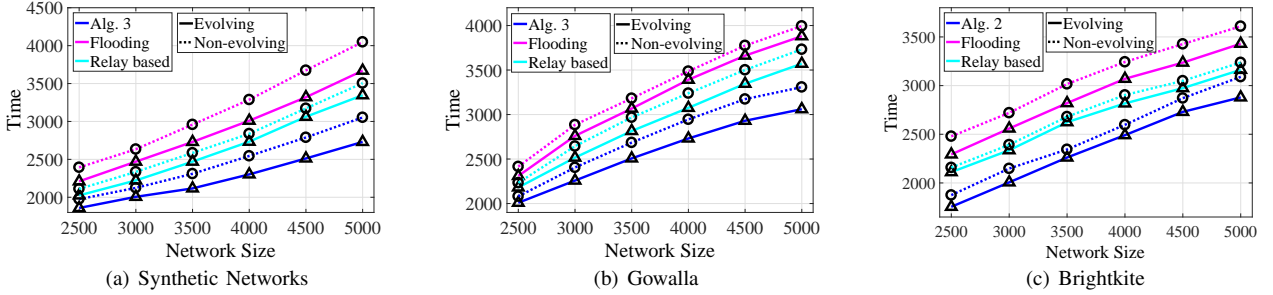


Fig. S2. Comparison of Transmission Time Under Random Walk Model.

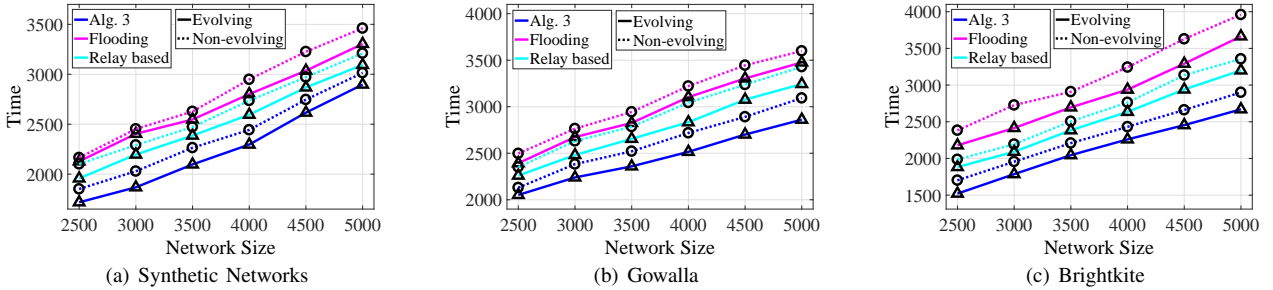


Fig. S3. Comparison of Transmission Time Under Random Waypoint Model.

$\Theta((n\sqrt{\log n})^{\frac{1}{3}})$. However, this goal is still achievable in our scheme by expanding the social depth to nearly $\frac{2D}{3}$, where no relay is applied.

From the above discussion, we can see that our proposed schemes could achieve favorable results compared with other schemes under same conditions, and even comparable results with some schemes that have additional aids.

F. Discussion on the Multi-File Case

Moreover, our scheme and results could be easily extended to the multi-file scenario with M files. In the static case, we ask each active user to maintain M request trees with each tree composed of requests for one type of content. And every M consecutive time slots aggregate to be a mega-slot. Each slot in the mega-slot is arranged to serve children in one type of request tree. As a result, the transmission time of each content is M times of the previous result, since the actual bit rate for certain content is M times smaller.

In the mobile case, the idea is similar. To accommodate the multi-file scenario (M files), we aggregate M consecutive slots to be a mega-slot. And each time slot in the mega slot is allocated for transmitting certain type of content. Since each type of content has to wait $M - 1$ times more slots to be

transmitted, the resultant transmission time of each content is M times of the previous result.

III. EXPERIMENTAL COMPARISON WITH BASELINES

Due to the space limitation, we move the results of comparison with baselines here. The results are presented in Fig. S1, Fig. S2 and Fig. S3.

[A1] F. Chung and L. Lu, "The average distance in a random graph with given expected degrees," *Internet Math.*, vol. 1, no. 1, pp. 91–113, 2004.

[A2] T. Stavros, and A. J. Goldsmith, "Large wireless networks under fading, mobility, and delay constraints," in *IEEE Conf. Comput. Commun., INFOCOM*, 2004, pp. 609–619 IEEE, 2004.

[A3] L. Xiaojun, and N. Shroff, "Towards achieving the maximum capacity in large mobile wireless networks under delay constraints," *J. of Commun. and Netw.*, vol. 6, no. 4, pp. 352–361, Dec. 2004.